JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON TUESDAY 28th JANUARY 2025) **TIME : 3:00 PM TO 6:00 PM**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Bag B_1 contains 6 white and 4 blue balls, Bag B_2 contains 4 white and 6 blue balls, and Bag B_3 contains 5 white and 5 blue balls. One of the bags is selected at random and a ball is drawn from it. If the ball is white, then the probability, that the ball is drawn from Bag B_2, is :

(1) $\frac{1}{3}$ (2) $\frac{4}{15}$

(3) $\frac{2}{3}$ (4) $\frac{2}{5}$

Ans. (2)

Sol. E_1 : Bag B_1 is selected

B_1	B_2	B_3
6W 4B	4W 6B	5W 5B

E_2 : bag B_2 is selected

E_3 : Bag B_3 is selected

A : Drawn ball is white

We have to find $P\left(\frac{E_2}{A}\right)$

$$P\left(\frac{E_2}{A}\right)=\frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$=\frac{\frac{1}{3}\times\frac{4}{10}}{\frac{1}{3}\times\frac{6}{10}+\frac{1}{3}\times\frac{4}{10}+\frac{1}{3}\times\frac{5}{10}}$$

$$=\frac{4}{15}$$

2. Let A, B, C be three points in xy-plane, whose position vector are given by $\sqrt{3}\hat{i}+\hat{j}$, $\hat{i}+\sqrt{3}\hat{j}$ and $a\hat{i}+(1-a)\hat{j}$ respectively with respect to the origin O. If the distance of the point C from the line bisecting the angle between the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is $\frac{9}{\sqrt{2}}$, then the sum of all the possible values of a is :

(1) 1 (2) 9/2

(3) 0 (4) 2

Ans. (1)

Sol. Equation of angle bisector : x – y = 0

$$\left|\frac{a(1-a)}{\sqrt{2}}\right|=\frac{9}{\sqrt{2}}\Rightarrow a=5 \text{ or } -4$$

Sum = 5 + (–4) = 1

3. If the components of $\vec{a}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ along and perpendicular to $\vec{b}=3\hat{i}+\hat{j}-\hat{k}$ respectively, are $\frac{16}{11}(3\hat{i}+\hat{j}-\hat{k})$ and $\frac{1}{11}(-4\hat{i}-5\hat{j}-17\hat{k})$, then $\alpha^2+\beta^2+\gamma^2$ is equal to :

(1) 23 (2) 18

(3) 16 (4) 26

Ans. (4)

Sol. let

$\vec{a}_{11}$ = component of $\vec{a}$ along $\vec{b}$

$\vec{a}_1$ = component of $\vec{a}$ perpendicular to $\vec{b}$

$$\vec{a}_{11}=\frac{16}{11}(3\hat{i}+\hat{j}-\hat{k})$$

$$\vec{a}_1=\frac{1}{11}(-4\hat{i}-5\hat{j}-17\hat{k})$$

$$\because\ \vec{a}=\vec{a}_{11}+\vec{a}_1$$

$$\therefore\ \vec{a}=\frac{16}{11}(3\hat{i}+\hat{j}-\hat{k})+\frac{1}{11}(-4\hat{i}-5\hat{j}-17\hat{k})$$

$$=\frac{44}{11}\hat{i}+\frac{11}{11}\hat{j}-\frac{33}{11}\hat{k}$$

$$\vec{a}=4\hat{i}+\hat{j}-3\hat{k}$$

$\alpha = 4$ $\beta = 1$ $\gamma = -3$

$\alpha^2+\beta^2+\gamma^2 = 16 + 1 + 9 = 26$

4. If $\alpha + i\beta$ and $\gamma + i\delta$ are the roots of $x^2-(3-2i)x-(2i-2)=0$, $i=\sqrt{-1}$, then $\alpha\gamma+\beta\delta$ is equal to :

(1) 6 (2) 2

(3) –2 (4) – 6

Ans. (2)

Sol. $x^2 - (3-2i)x - (2i-2) = 0$

$$x = \frac{(3-2i) \pm \sqrt{(3-2i)^2 - 4(1)(-(2i-2))}}{2(1)}$$

$$= \frac{(3-2i) \pm \sqrt{9-4-12i+8i-8}}{2}$$

$$= \frac{3-2i \pm \sqrt{-3-4i}}{2}$$

$$= \frac{3-2i \pm \sqrt{(1)^2 + (2i)^2 - 2(1)(2i)}}{2}$$

$$= \frac{3-2i \pm (1-2i)}{2}$$

$$\Rightarrow \frac{3-2i+1-2i}{2} \text{ or } \frac{3-2i-1+2i}{2}$$

$\Rightarrow 2-2i$ or $1+0i$

So $\alpha\gamma + \beta\delta = 2(1) + (-2)(0) = 2$

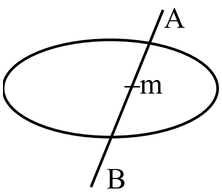
5. If the midpoint of a chord of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is $(\sqrt{2}, 4/3)$, and the length of the chord is $\frac{2\sqrt{\alpha}}{3}$, then α is :

(1) 18 (2) 22

(3) 26 (4) 20

Ans. (2)

Sol.



If $m\left(\sqrt{2}, \frac{4}{3}\right)$ than equation of of AB is

$T = S_1$

$$\frac{x\sqrt{2}}{9} + \frac{y}{4}\left(\frac{4}{3}\right) = \frac{(\sqrt{2})^2}{9} + \frac{\left(\frac{4}{3}\right)^2}{4}$$

$$\frac{\sqrt{2}x}{9} + \frac{y}{3} = \frac{2}{9} + \frac{4}{9}$$

$$\sqrt{2}x + 3y = 6 \Rightarrow y = \frac{6-\sqrt{2}x}{3} \text{ put in ellipse}$$

$$\text{So, } \frac{x^2}{9} + \frac{(6-\sqrt{2}x)^2}{9 \times 4} = 1$$

$$4x^2 + 36 + 2x^2 - 12\sqrt{2}x = 36$$

$$6x^2 - 12\sqrt{2}x = 0$$

$$6x\left(x - 2\sqrt{2}\right) = 0$$

$x = 0$ & $x = 2\sqrt{2}$

So $y = 2$ $\quad y = \frac{2}{3}$

$$\text{Length of chord} = \sqrt{\left(2\sqrt{2}-0\right)^2 + \left(\frac{2}{3}-2\right)^2}$$

$$= \sqrt{8 + \frac{16}{9}}$$

$$= \sqrt{\frac{88}{9}} = \frac{2}{3}\sqrt{22} \text{ so } \boxed{\alpha = 22}$$

6. Let S be the set of all the words that can be formed by arranging all the letters of the word GARDEN. From the set S, one word is selected at random. The probability that the selected word will NOT have vowels in alphabetical order is :

(1) $\frac{1}{4}$ (2) $\frac{2}{3}$

(3) $\frac{1}{3}$ (4) $\frac{1}{2}$

Ans. (4)

Sol. A, E,G R D N

$$\text{Probabllity (P)} = \frac{\text{favourable case}}{\text{Total case}}$$

(when A & E are in order)

Total case = 6!

Favourable case $= {}^6C_2 \cdot 4!$

$$P = \frac{(15)4!}{(30)4!}$$

$$\text{Probablity when not in order} = 1 - \frac{1}{2} = \frac{1}{2}$$

7. Let f be a real valued continuous function defined on the positive real axis such that $g(x) = \int_0^x t f(t) dt$. If $g(x^3) = x^6 + x^7$, then value of $\sum_{r=1}^{15} f(r^3)$ is :

(1) 320 (2) 340
(3) 270 (4) 310

Ans. (4)

Sol. $g(x) = x2 + x^{\frac{7}{3}}$

$g'(x) = 2x + \frac{7}{3}x^{\frac{4}{3}}$

$f(x) = \frac{g'(x)}{x}$

$f(x) = 2 + \frac{7}{3}x^{\frac{1}{3}}$

$f(r^3) = 2 + \frac{7r}{3}$

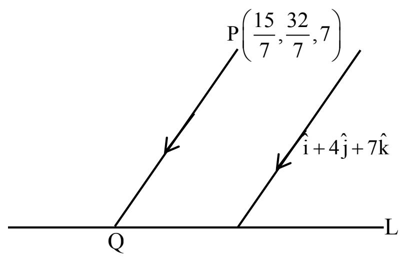
$\sum_{r=1}^{15}\left(1 + \frac{7}{3}r\right) = 310$

8. The square of the distance of the point $\left(\frac{15}{7}, \frac{32}{7}, 7\right)$ from the line $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ in the direction of the vector $\hat{i} + 4\hat{j} + 7\hat{k}$ is :

(1) 54 (2) 41
(3) 66 (4) 44

Ans. (3)

Sol.



$L = \frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$

$PQ = \frac{x - \frac{15}{7}}{1} = \frac{y - \frac{32}{7}}{4} = \frac{z-7}{7} = \lambda$

$\Rightarrow Q\left(\lambda + \frac{15}{7}, 4\lambda + \frac{32}{7}, 7\lambda + 7\right)$

Since Q lies on line L

So, $\frac{\lambda + \frac{15}{7} + 1}{3} = \frac{7\lambda + 7 + 5}{7}$

$\Rightarrow 7\lambda + 22 = 21\lambda + 36$

$\Rightarrow \lambda = -1$

$\therefore$ Point $Q\left(\frac{8}{7}, \frac{4}{7}, 0\right)$

$PQ = \sqrt{\left(\frac{15}{7} - \frac{8}{7}\right)^2 + \left(\frac{32}{7} - \frac{4}{7}\right)^2 + (7-0)}$

$PQ = \sqrt{66}$

$\Rightarrow (PQ)^2 = 66$

9. The area of the region bounded by the curves $x(1 + y^2) = 1$ and $y^2 = 2x$ is :

(1) $2\left(\frac{\pi}{2} - \frac{1}{3}\right)$ (2) $\frac{\pi}{4} - \frac{1}{3}$

(3) $\frac{\pi}{2} - \frac{1}{3}$ (4) $\frac{1}{2}\left(\frac{\pi}{2} - \frac{1}{3}\right)$

Ans. (3)

Sol. $x(1 + y^2) = 1$ ..... (1)

$y^2 = 2x$ ..... (2)

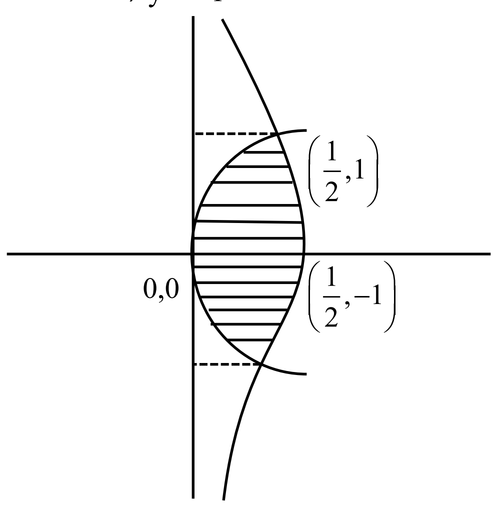
From equation (1) & (2)

$x(1 + 2x) = 1 \Rightarrow 2x^2 + x - 1 = 0$

$\Rightarrow x = \frac{1}{2}$, $x = -1$ (Reject)

$\Rightarrow y^2 = 2\left(\frac{1}{2}\right)$

$\Rightarrow y = \pm 1$



Area bounded = $\int_{-1}^{1}\left(\frac{1}{1+y^2}-\frac{y^2}{2}\right)dy$

$= \left(\tan^{-1}y-\frac{y^3}{6}\right)\Big|_{-1}^{1}$

$= \frac{\pi}{2}-\frac{1}{3}$

10. Let $A=\begin{bmatrix}\frac{1}{\sqrt{2}} & -2\\ 0 & 1\end{bmatrix}$ and $P=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}, \theta>0.$ If $B = PAP^T$, $C = P^TB^{10}P$ and the sum of the diagonal elements of C is $\frac{m}{n}$, where gcd(m, n) = 1, then m + n is :

(1) 65 (2) 127

(3) 258 (4) 2049

Ans. (1)

Sol. $P=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$

$\because P^TP = I$

$B = PAPT$

Pre multiply by P^T (Given)

$P^TB = P^TP\ AP^T = AP^T$

Now post multiply by P

$P^TBP = AP^TP = A$

So $A^2 = \underbrace{P^TBP\ \ P^T}_{I}\ BP$

$A^2 = P^TB^2P$

Similarly $A^{10} = P^TB^{10}P = C$

$A=\begin{bmatrix}\frac{1}{\sqrt{2}} & -2\\ 0 & 1\end{bmatrix}$ (Given)

$\Rightarrow A^2=\begin{bmatrix}\frac{1}{2} & -\sqrt{2}-2\\ 0 & 1\end{bmatrix}$

Similarly check A^3 and so on since $C = A^{10}$

$\Rightarrow$ Sum of diagonal elements of C is $\left(\frac{1}{\sqrt{2}}\right)^{10}+1$

$= \frac{1}{32}+1=\frac{33}{32}=\frac{m}{n}$

g cd(m,n) = 1 (Given)

$\Rightarrow m + n = 65$

11. If $f(x)=\int\frac{1}{x^{1/4}(1+x^{1/4})}dx$, $f(0) = -6$, then $f(1)$ is equal to :

(1) $\log_e 2+2$ (2) $4(\log_e 2-2)$

(3) $2-\log_e 2$ (4) $4(\log_e 2+2)$

Ans. (1)

Sol. let $x = t^4$

$dx = 4t^3\ dt$

then $\int\frac{1}{x^{\frac{1}{4}}\left(1+x^{\frac{1}{4}}\right)}dx \Rightarrow \int\frac{4t^3dt}{t(1+t)}$

$\Rightarrow \int\frac{4t}{1+t}dt \Rightarrow 4\int\frac{(t^2-1)+1}{1+t}dt$

$\Rightarrow 4\int(t-1)+\frac{1}{t+1}dt$

$\Rightarrow 4\left\{\frac{(t-1)^2}{2}+\ell n(t+1)\right\}+c$

hence $f(x) = 2\left(x^{\frac{1}{4}}-1\right)^2+4\ell n\left(1+x^{\frac{1}{4}}\right)+c$

$f(0) = -6 \Rightarrow 2 + 4\ell n + 6 = -6 \rightarrow C = -8$

now $f(1) = 4\ell n\ 2 - 8$

$= 4(\ell n2-2)$

12. Let f : R → R be a twice differentiable function such that $f(2)$ = 1. If F(x) = $xf(x)$ for all x ∈ R, $\int_0^2 xF'(x)dx=6$ and $\int_0^2 x^2F''(x)dx=40$, then $F'(2)+\int_0^2 F(x)dx$ is equal to :

(1) 11 (2) 15

(3) 9 (4) 13

Ans. (2)

Sol. $\int_0^2 xF'(x)dx = 6$

$= xF(x)\big|_0^2 - \int_0^2 f(x)dx = 6$

$= 2F(2) - \int_0^2 xF(x)dx = 6$ [$\therefore$ f(2) = 2F(2) = 2]

$\int_0^2 xF(x)dx = -2$ ... (1)

$\Rightarrow \int_0^2 F(x)dx = -2$ ... (2)

Also

$\int_0^2 x^2F''(x)dx = x^2F'(x)\big|_0^2 - 2\int_0^2 xF'(x)dx = 40$

$= 4F'(2) - 2 \times 6 = 40$

$F'(2) = 13$

$\therefore F'(2) + \int_0^2 F(x) = 13 - 2 = 11$

13. For positive integers n, if $4a_n = (n^2 + 5n + 6)$ and $S_n = \sum_{k=1}^{n}\left(\frac{1}{a_k}\right)$, then the value of 507 S_{2025} is :

(1) 540 (2) 1350

(3) 675 (4) 135

Ans. **(3)**

Sol. $a_n = \frac{n^2 + 5n + 6}{4}$

$S_n = S_n = \sum_{k=1}^{n}\frac{1}{a_k} = \sum_{1}^{n}\frac{4}{k^2 + 5k + 6}$

$= 4\sum_{k=1}^{n}\frac{1}{(k+2)(k+3)}$

$= 4\sum_{k=1}^{n}\frac{1}{k+2} - \frac{1}{k+3}$

$= 4\left(\frac{1}{3} - \frac{1}{4}\right) + 4\left(\frac{1}{4} - \frac{1}{5}\right) + \ldots\ldots$

$4\left(\frac{1}{n+2} - \frac{1}{n+3}\right)$

$= 4\left(\frac{1}{3} - \frac{1}{n+3}\right)$

$= \frac{4n}{3(n+3)}$

$507\, S_{2025} = \frac{(507)(4)(2025)}{3(2028)}$

$= 675$

14. Let $f : [0, 3] \to A$ be defined by $f(x) = 2x^3 - 15x^2 + 36x + 7$ and $g : [0, \infty) \to B$ be defined by $g(x) = \frac{x^{2025}}{x^{2025} + 1}$. If both the functions are onto and S = {x ∈ **Z** : x ∈ A or x ∈ B}, then n (S) is equal to :

(1) 30 (2) 36

(3) 29 (4) 31

Ans. **(1)**

Sol. as f(x) is onto hence A is range of f(x)

now $f'(x) = 6x^2 - 30x + 36$

$= 6(x-2)(x-3)$

$f(2) = 16 - 60 + 72 + 7 = 35$

$f(3) = 54 - 135 + 108 + 7 = 34$

$f(0) = 7$

hence range $\in [7, 35] = A$

also for range of g (x)

$g(x) = 1 - \frac{1}{x^{2025} + 1} \in [0, 1) = B$

s = {0, 7,8, ...35} hence n(s) = 30

15. Let [x] denote the greatest integer less than or equal to x. Then domain of $f(x) = \sec^{-1}(2[x]+1)$ is :

(1) $(-\infty, -1] \cup [0, \infty)$

(2) $(-\infty, -\infty)$

(3) $(-\infty, -1] \cup [1, \infty)$

(4) $(-\infty, \infty] - \{0\}$

Ans. **(2)**

Sol. $2[x] + 1 \le -1$ or $2[x] + 1 \ge 1$

$\Rightarrow [x] \le -1 \cup [x] \ge 0$

$\Rightarrow x \in (-\infty, 0) \cup x \in [0, \infty)$

$\Rightarrow x \in (-\infty, \infty)$

16. If $\sum_{r=1}^{13}\left\{\frac{1}{\sin\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)}\right\}=a\sqrt{3}+b$,

a, b ∈ **Z**, then $a^2 + b^2$ is equal to :

(1) 10 (2) 2

(3) 8 (4) 4

Ans. (3)

Sol. $\frac{1}{\sin\frac{\pi}{6}}\sum_{r=1}^{13}\frac{\sin\left[\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)-\left(\frac{\pi}{4}\right)-(r-1)\frac{\pi}{6}\right]}{\sin\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)}$

$\frac{1}{\sin\frac{\pi}{6}}\sum_{r=1}^{13}\left(\cot\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)\right)$

$= 2\sqrt{3}-2=\alpha\sqrt{3}+b$

So $a^2 + b^2 = 8$

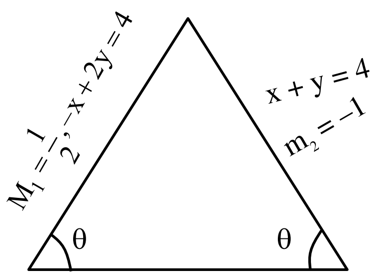
17. Two equal sides of an isosceles triangle are along $-x + 2y = 4$ and $x + y = 4$. If m is the slope of its third side, then the sum, of all possible distinct values of m, is :

(1) –6 (2) 12

(3) 6 (4) $-2\sqrt{10}$

Ans. (3)

Sol.



$\tan\theta=\frac{m-\frac{1}{2}}{1+\frac{1}{2}.m}=\frac{-1-m}{1-m}=\frac{m+1}{m-1}$

$\frac{2m-1}{2+m}=\frac{m+1}{m-1}$

$2m^2 - 3m + 1 = m^2 + 3m + 2$

$m^2 - 6m - 1 = 0$

sum of root = 6

sum is 6

18. Let the coefficients of three consecutive terms T_r, T_{r+1} and T_{r+2} in the binomial expansion of $(a + b)^{12}$ be in a G.P. and let p be the number of all possible values of r. Let q be the sum of all rational terms in the binomial expansion of $\left(\sqrt[4]{3}+\sqrt[3]{4}\right)^{12}$. Then p + q is equal to :

(1) 283 (2) 295

(3) 287 (4) 299

Ans. (1)

Sol. $(a+b)^{\frac{1}{2}}$

$T_r, T_{r+1}, T_{r+2} \to GP$

So, $\frac{T_{r+1}}{T_r}=\frac{T_{r+2}}{T_{r+1}}$

$\frac{^{12}C_r}{^{12}C_{r-1}}=\frac{^{12}C_{r+1}}{^{12}C_r}$

$\frac{12-r+1}{r}=\frac{12-(r+1)+1}{r+1}$

$(13 - r)(r + 1) = (12 - r)(r)$

$-r + 12r + 13 = 12r - r^2$

$13 = 0$

No value of r possible

So P = 0

$\left(3^{\frac{1}{4}}+4^{\frac{1}{3}}\right)^{12}=\sum {}^{12}C_r\left(3^{\frac{1}{4}}\right)^{12-r}\left(4^{\frac{1}{3}}\right)^r$

Exponent of $\left(3^{\frac{1}{4}}\right)$	exponent of $\left(4^{\frac{1}{3}}\right)$	term
12	0	27
0	12	256

q = 27 + 256 = 283

p + q = 0 + 283 = 283

19. If A and B are the points of intersection of the circle $x^2 + y^2 - 8x = 0$ and the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ and a point P moves on the line $2x - 3y + 4 = 0$, then the centroid of ΔPAB lies on the line :

(1) $4x - 9y = 12$

(2) $x + 9y = 36$

(3) $9x - 9y = 32$

(4) $6x - 9y = 20$

Ans. **(4)**

Sol. $x^2 + y^2 - 8x = 0, \frac{x^2}{9} - \frac{y^2}{4} = 1$ .... (1)

$4x^2 - 9y^2 = 36$ ... (2)

Solve (1) & (2)

$4x^2 - 9(8x - x^2) = 36$

$13x^2 - 72x - 36 = 0$

$(13x + 6)(x = 6) = 0$

$x = \frac{-6}{13}, x = 6$

$x = \frac{-6}{13}$ (rejected)

$y \to$ Imaginary

$n = 6, \frac{36}{9} - \frac{y^2}{4} = 1$

$y^2 = 12, y = I\sqrt{12}$

$A(6, \sqrt{12}), B(6, -\sqrt{12})$

$p\left(\alpha, \frac{2\alpha + 4}{3}\right)$ P lies on

centroid (h,k) $2x - 3y + y = 0$

$h = \frac{12 + \alpha}{3}$, $\alpha = 3h - 12$

$k = \frac{\frac{2\alpha + 4}{3}}{3} \Rightarrow 2\alpha + 4 = 9k$

$\alpha = \frac{9k - 4}{2}$

$6h - 2y = 9k - 4$

$6x - 9y = 20$

20. Let $f : \mathbf{R} - \{0\} \to (-\infty, 1)$ be a polynomial of degree 2, satisfying $f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$. If $f(K) = -2K$, then the sum of squares of all possible values of K is :

(1) 1 (2) 6

(3) 7 (4) 9

Ans. **(2)**

Sol. as f(x) is a polynomial of degree two let it be

$f(x) = ax^2 + bx + c \quad (a \neq 0)$

on satisfying given conditions we get

$C = 1$ & $a = \pm 1$

hence $f(x) = 1 \pm x^2$

also range $\in (-\infty, 1]$ hence

$f(x) = 1 - x^2$

now $f(k) = -2k$

$1 - k^2 = -2k \to k^2 - 2k - 1 = 0$

let roots of this equation be α & β

then $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 4 - 2(-1) = 6$

SECTION-B

21. The number of natural numbers, between 212 and 999, such that the sum of their digits is 15, is ______.

Ans. **(64)**

Sol.

x	y	z

Let $x = 2 \Rightarrow y + z = 13$

(4,9), (5,8), (6,7), (7,6), (8,5), (9,4), $\to$ 6

Let $x = 3 \to y + z = 12$

(3,9), (4,8), ..., (9,3) $\to$7

Let $x = 4 \to y + z = 11$

(2,9), (3,8), ..., (9,1) $\to$9

Let $x = 5 \to y + z = 10$

(1,9), (2,8), ..., (9,1) $\to$10

Let $x = 6 \to y + z = 9$

(0,9), (1,8), ..., (9,0) $\to$9

Let $x = 7 \to y + z = 8$

(0,9), (1,7), ..., (8,0) $\to$9

Let $x = 8 \to y + z = 7$

(0,7), (1,6), ..., (7,0) $\to$8

Let $x = 9 \to y + z = 6$

(0,6), (1,5), ..., (6,0) $\to$7

Total = 6 = 7 + 8 + 9 + 10 + 9 + 8 + 7 = 64

22. Let $f(x)=\lim\limits_{n\to\infty}\sum\limits_{r=0}^{n}\left(\dfrac{\tan(x/2^{r+1})+\tan^3(x/2^{r+1})}{1-\tan^2(x/2^{r+1})}\right)$.

Then $\lim\limits_{x\to 0}\dfrac{e^x-e^{f(x)}}{(x-f(x))}$ is equal to ____.

Ans. (1)

Sol. $f(x)=\lim\limits_{n\to\infty}\sum\limits_{r=0}^{n}\left(\tan\dfrac{x}{2^r}-\tan\dfrac{x}{2^{r+1}}\right)=\tan x$

$$\lim_{x\to 0}\left(\frac{e^x-e^{\tan x}}{x-\tan x}\right)=\lim_{x\to 0}e^{\tan x}\frac{\left(e^{x-\tan x}-1\right)}{\left(x-\tan x\right)}$$

$=1$

23. The interior angles of a polygon with n sides, are in an A.P. with common difference 6°. If the largest interior angle of the polygon is 219°, then n is equal to _____.

Ans. (20)

Sol. $\dfrac{n}{2}\left(2a+(n-1)6\right)=(n-2).180°$

$an + 3n^2 - 3n = (n-2).\ 180°$ ...(1)

Now according to question

$a + (n-1)6° = 219°$

$\Rightarrow a = 225° - 6n°$ ...(2)

Putting value of a from equation (2) in (1)

We get

$(225n - 6n^2) + 3n^2 - 3n = 180n - 360$

$\Rightarrow 2n^2 - 42n - 360 = 0$

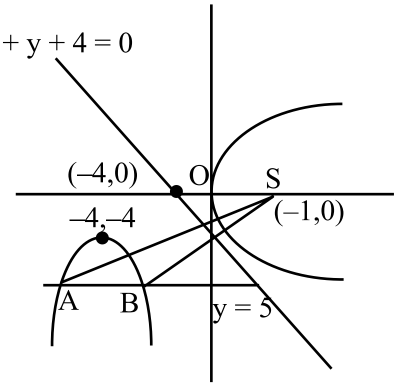
$\Rightarrow n2 - 14n - 120 = 0$

$n = 20, -6$(rejected)

24. Let A and B be the two points of intersection of the line y + 5 = 0 and the mirror image of the parabola $y^2 = 4x$ with respect to the line x + y + 4 = 0. If d denotes the distance between A and B, and a denotes the area of ΔSAB, where S is the focus of the parabola $y^2 = 4x$, then the vlaue of (a + d) is ____.

Ans. (14)

Sol.



$\text{Area} = \dfrac{1}{2}\times 4\times 5 = 10 = a$

$6 = 4$

So $a + d = 14$

25. If y = y(x) is the solution of the differential equation,

$$\sqrt{4-x^2}\frac{dy}{dx}=\left(\left(\sin^{-1}\left(\frac{x}{2}\right)\right)^2-y\right)\sin^{-1}\left(\frac{x}{2}\right),$$

$-2\le x\le 2,\ y(2)=\left(\dfrac{\pi^2-8}{4}\right)$, then $y^2(0)$ is equal to ____.

Ans. (4)

Sol. $\dfrac{dy}{dx}+\dfrac{\left(\sin^{-1}\dfrac{x}{2}\right)}{\sqrt{4-x^2}}y=\dfrac{\left(\sin^{-3}\dfrac{x}{2}\right)^3}{\sqrt{4-x^2}}$

$$ye^{\frac{\left(\sin^{-1}\frac{x}{2}\right)^2}{2}}=\int\frac{\left(\sin^{-3}\frac{x}{2}\right)^3}{4-x^2}e^{\frac{\left(\sin^{-1}\frac{x}{2}\right)^2}{2}}dx$$

$$y=\left(\sin^{-1}\frac{x}{2}\right)^2-2+c.e^{-\frac{\left(\sin^{-1}\frac{x}{2}\right)^2}{2}}$$

$y(2)=\dfrac{\pi^2}{4}-2\Rightarrow c=0$

$y\,(0) = -2$