# JEE–MAIN EXAMINATION – JANUARY 2025

**(HELD ON FRIDAY 24th JANUARY 2025)** **TIME : 3:00 PM TO 6:00 PM**

**MATHEMATICS** **TEST PAPER WITH SOLUTION**

## SECTION-A

**1.** The equation of the chord, of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, whose mid-point is (3,1) is :

(1) 48x + 25y = 169 (2) 4x + 122y = 134

(3) 25x + 101y = 176 (4) 5x + 16y = 31

**Ans.** **(1)**

**Sol.** Equation of chord with given middle point

$T = S_1$

$\Rightarrow \frac{3x}{25}+\frac{y}{16}-1=\frac{9}{25}+\frac{1}{16}-1$

48x + 25y = 144 + 25

48x + 25y = 169 Ans.

**2.** The function $f$ : $(-\infty, \infty) \rightarrow (-\infty, 1)$, defined by $f(x)=\frac{2^x-2^{-x}}{2^x+2^{-x}}$ is :

(1) One-one but not onto

(2) Onto but not one-one

(3) Both one-one and onto

(4) Neither one-one nor onto

**Ans.** **(1)**

**Sol.** $f(x) = \frac{2^{2x}-1}{2^{2x}+1}$

$=1-\frac{2}{2^{2x}+1}$

$f'(x) = \frac{2}{\left(2^{2x}+1\right)^2}.2.2^{2x}.\ell n2$ i.e always +ve

so f(x) is ↑ function

$\therefore f(-\infty) = -1$

$f(\infty) = 1$

$\therefore f(x) \in (-1, 1) \neq$ co-domain

so function is one-one but not onto

**3.** If $\alpha > \beta > \gamma > 0$, then the expression

$$\cot^{-1}\left\{\beta+\frac{(1+\beta^2)}{(\alpha-\beta)}\right\}+\cot^{-1}\left\{\gamma+\frac{(1+\gamma^2)}{(\beta-\gamma)}\right\}+\cot^{-1}\left\{\alpha+\frac{(1+\alpha^2)}{(\gamma-\alpha)}\right\}$$ is equal to:

(1) $\frac{\pi}{2}-(\alpha+\beta+\gamma)$ (2) $3\pi$

(3) 0 (4) $\pi$

**Ans.** **(4)**

**Sol.** $\Rightarrow \cot^{-1}\left(\frac{\alpha\beta+1}{\alpha-\beta}\right)+\cot^{-1}\left(\frac{\beta\gamma+1}{\beta-\gamma}\right)+\cot^{-1}\left(\frac{\alpha\gamma+1}{\gamma-\alpha}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\alpha-\beta}{1+\alpha\beta}\right)+\tan^{-1}\left(\frac{\beta-\gamma}{1+\beta\gamma}\right)+\pi+\tan^{-1}\left(\frac{\gamma-\alpha}{1+\gamma\alpha}\right)$

$\Rightarrow\left(\tan^{-1}\alpha-\tan^{-1}\beta\right)+\left(\tan^{-1}\beta-\tan^{-1}\gamma\right)+\left(\pi+\tan^{-1}\gamma-\tan^{-1}\alpha\right)$

$\Rightarrow \pi$

**4.** Let f : $(0, \infty) \rightarrow$ **R** be a function which is differentiable at all points of its domain and satisfies the condition $x^2f'(x) = 2xf(x) + 3$, with f(1) = 4. Then 2f(2) is equal to:

(1) 29 (2) 19

(3) 39

(4) 23

**Ans.** **(3)**

**Sol.** $x^2f'(x) - 2x f(x) = 3$

$\left(\frac{x^2f'(x)-2xf(x)}{\left(x^2\right)^2}\right)=\frac{3}{\left(x^2\right)^2}$

$\Rightarrow \frac{d}{dx}\left(\frac{f(x)}{x^2}\right)=\frac{3}{x^4}$

Integrating both sides

$\frac{f(x)}{x^2}=-\frac{1}{x^3}+C$

$f(x) = -\frac{1}{x}+Cx^2$

put x = 1

$4 = -1 + C \Rightarrow C = 5$

$f(x) = -\frac{1}{x}+5x^2$

Now $2 \times f(2) = 2\times\left[-\frac{1}{2}+5\times2^2\right]$

= 39

**5.** Let

$$A=\left\{x\in(0,\pi)-\left\{\frac{\pi}{2}\right\}:\log_{(2/\pi)}|\sin x|+\log_{(2/\pi)}|\cos x|=2\right\}$$

and

$B=\left\{x\geq 0:\sqrt{x}(\sqrt{x}-4)-3\left|\sqrt{x}-2\right|+6=0\right\}$. Then

$n(A\cup B)$ is equal to:

(1) 4 (2) 2

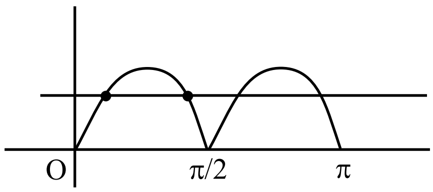
(3) 8 (4) 6

**Ans.** **(3)**

**Sol.** A : $\log_{2/\pi}|\sin x| + \log_{2/\pi}|\cos x| = 2$

$\Rightarrow \log_{2/\pi}(|\sin x \cdot \cos x|) = 2$

$\Rightarrow |\sin 2x| = \dfrac{8}{\pi^2}$




Number of solution 4

B : let $\sqrt{x} = t < 2$

Then $\sqrt{x}\left(\sqrt{x}-4\right)+3\left(\sqrt{x}-2\right)+6=0$

$\Rightarrow t^2 - 4t + 3t - 6 + 6 = 0$

$\Rightarrow t^2 - t = 0,\ t = 0,\ t = 1$

$x = 0,\ x = 1$

again let $\sqrt{x} = t > 2$

then $t^2 - 4t - 3t + 6 + 6 = 0$

$\Rightarrow t^2 - 7t + 12 = 0$

$\Rightarrow t = 3, 4$

$x = 9, 16$

Total number of solutions

$n(A\cup B) = 4 + 4 = 8$

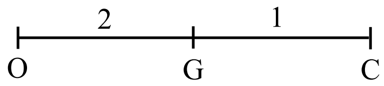
**6.** Let the position vectors of three vertices of a triangle be $4\vec{p}+\vec{q}-3\vec{r},\ -5\vec{p}+\vec{q}+2\vec{r}$ and $2\vec{p}-\vec{q}+2\vec{r}$. If the position vectors of the orthocenter and the circumcenter of the triangle are $\dfrac{\vec{p}+\vec{q}+\vec{r}}{4}$ and $\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r}$ respectively, then $\alpha+2\beta+5\gamma$ is equal to:

(1) 3 (2) 1

(3) 6 (4) 4

**Ans.** **(1)**

**Sol.** We know that




O (orthocentre) $\dfrac{\vec{p}+\vec{q}+\vec{r}}{4}$

C (circum centre) $\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r}$

C (centroid) $= \dfrac{\vec{p}+\vec{q}+\vec{r}}{3}$

by relation

$\Rightarrow 2\left(\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r}\right)+\dfrac{\vec{p}+\vec{q}+\vec{r}}{4}=3\left(\dfrac{\vec{p}+\vec{q}+\vec{r}}{3}\right)$

$\Rightarrow 8\left(\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r}\right)=3\left(\vec{p}+\vec{q}+\vec{r}\right)$

$\Rightarrow 8\alpha = 3,\ 8\beta = 3,\ 8\gamma = 3$

$\alpha = \dfrac{3}{8},\ \beta = \dfrac{3}{8},\ \gamma = \dfrac{3}{8}$

$\therefore\ \alpha + 2\beta + 3\gamma$

$\dfrac{3}{8}+\dfrac{6}{8}+\dfrac{15}{8}=\dfrac{24}{8}=3$

**7.** Let [x] denote the gereatest integer function, and let m and n respectively be the numbers of the points, where the function $f(x) = [x] + |x - 2|$, $-2 < x < 3$, is not continuous and not differentiable. Then m + n is equal to:

(1) 6 (2) 9

(3) 8 (4) 7

**Ans.** **(3)**

**Sol.** $f(x) = [x] + |x - 2| \quad -2 < x < 3$

$$f(x)=\begin{cases}-x, & -2<x<-1\\ -x+1, & -1\leq x<0\\ -x+2, & 0\leq x<1\\ -x+3, & 1\leq x<2\\ x, & 2\leq x<3\end{cases}$$

So f(x) is not continuous at 4 points and not differentiable at 4 point

So m + n = 4 + 4 = 8

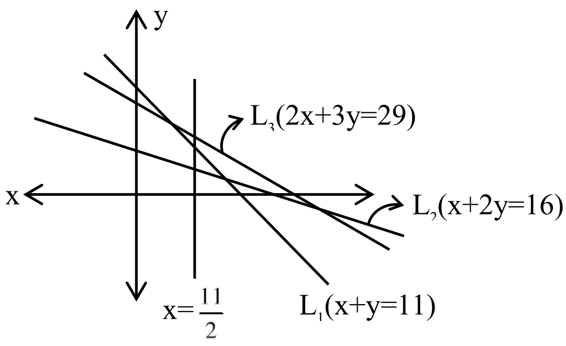
**8.** Let the points $\left(\frac{11}{2}, \alpha\right)$ lie on or inside the triangle with sides x + y = 11, x + 2y = 16 and 2x + 3y = 29. Then the product of the smallest and the largest values of α is equal to :

(1) 22 (2) 44

(3) 33 (4) 55

**Ans.** **(3)**

**Sol.**




Point of intersection of $x = \frac{11}{2}$ with $L_1$ & $L_3$

gives, $\alpha_{min} = \frac{11}{2}$

and $\alpha_{max} = 6$

$\therefore \alpha_{min} \cdot \alpha_{max} = \frac{11}{2} \times 6 = 33$

**9.** In an arithmetic progression, if $S_{40}$ = 1030 and $S_{12}$ = 57, then $S_{30} - S_{10}$ is equal to:

(1) 510 (2) 515

(3) 525 (4) 505

**Ans.** **(2)**

**Sol.** Let a & d are first term and common diff of an AP.

$S_{40} = \frac{40}{2}[2a + 39d] = 1030$ ....(1)

$S_{12} = \frac{12}{2}[2a + 11d] = 57$ ....(2)

by (1) & (2)

$a = -\frac{7}{2}$ $d = \frac{3}{2}$

$\therefore S_{30} - S_{10} = \frac{30}{2}[2a + 29d] - \frac{10}{2}[2a + 9d]$

$= 20a - 390d$

$= 515$

**10.** If $7 = 5 + \frac{1}{7}(5 + \alpha) + \frac{1}{7^2}(5 + 2\alpha) + \frac{1}{7^3}(5 + 3\alpha) +$ ....... ∞, then the value of α is:

(1) 1 (2) $\frac{6}{7}$

(3) 6 (4) $\frac{1}{7}$

**Ans.** **(3)**

**Sol.** Let $S = 5 + \frac{1}{7}(5 + \alpha) + \frac{1}{7^2}(5 + 2\alpha) + \ldots$

$\frac{1}{7}S = \frac{1}{7}(5) + \frac{1}{7^2}(5 + \alpha) + \ldots \infty$

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$\frac{6}{7}(S) = 5 + \frac{1}{7}\alpha\left(\frac{1}{1 - \frac{1}{7}}\right)$

$6 = 5 + \frac{\alpha}{6} \Rightarrow \alpha = 6$

**11.** If the system of equations

x + 2y – 3z = 2

2x + λy + 5z = 5

14x + 3y + μz = 33

has infinitely many solutions, then λ + μ is equal to:

(1) 13 (2) 10

(3) 11 (4) 12

**Ans.** **(4)**

**Sol.** $D = \begin{vmatrix} 1 & 2 & -3 \\ 2 & \lambda & 5 \\ 14 & 3 & \mu \end{vmatrix} = 0, \lambda\mu + 42\lambda - 4\mu + 107 = 0$

$D_1 = 2\lambda\mu + 99\lambda - 10\mu + 255$

$D_2 = 13 - \mu$

$D_3 = 5\lambda + 5$

$D_2 = 0 \Rightarrow \mu = 13$ & $D_3 = 0 \Rightarrow \lambda = -1$

check & verify for these values D & $D_2 = 0$

**12.** Let (2, 3) be the largest open interval in which the function $f(x) = 2\log_e(x - 2) - x^2 + ax + 1$ is strictly increasing and (b, c) be the largest open interval, in which the function $g(x) = (x - 1)^3(x + 2 - a)^2$ is strictly decreasing. Then 100(a + b – c) is equal to:

(1) 280 (2) 360

(3) 420 (4) 160

**Ans.** **(2)**

**Sol.** $f'(x) = \frac{2}{x-2} - 2x + a \geq 0$

$f''(x) = \frac{-2}{(x-2)^2} - 2 < 0$

$f'(x) \downarrow$

$f'(3) \geq 0$

$2 - 6 + a \geq 0$

$a \geq 4$

$a_{min} = 4$

$g(x) = (x-1)^3 (x+2-a)^2$

$g(x) = (x-1)^3 (x-2)^2$

$g'(x) = (x-1)^3\, 2(x-2) + (x-2)^2\, 3(x-1)^2$

$= (x-1)^2 (x-2)(2x - 2 + 3x - 6)$

$= (x-1)^2 (x-2)(5x-8) < 0$

$x \in \left(\frac{8}{5}, 2\right)$

$100(a+b-c) = 100\left(4 + \frac{8}{5} - 2\right) = 360$

**13.** Suppose A and B are the coefficients of $30^{th}$ and $12^{th}$ terms respectively in the binomial expansion of $(1+x)^{2n-1}$. If 2A = 5B, then n is equal to:

(1) 22 (2) 21

(3) 20 (4) 19

**Ans. (2)**

**Sol.** $A = {}^{2n-1}C_{29} \quad B = {}^{2n-1}C_{11}$

$2\ {}^{2n-1}C_{29} = 5\ {}^{2n-1}C_{11}$

$2\frac{(2n-1)!}{29!(2n-30)!} = 5\frac{(2n-1)!}{(2n-12)!11!}$

$\frac{1}{29 \ldots 12 \cdot 5} = \frac{1}{(2n-12)(2n-13)\ldots(2n-29)2}$

$\frac{1}{30 \cdot 29 \ldots 12} = \frac{1}{(2n-12)(2n-13)\ldots(2n-29)12}$

$2n - 12 = 30$

$n = 21$

**14.** Let $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = \vec{a} \times (\hat{i} - 2\hat{k})$ and $\vec{c} = \vec{b} \times \hat{k}$. Then the projection of $\vec{c} - 2\hat{j}$ on $\vec{a}$ is:

(1) $3\sqrt{7}$

(2) $\sqrt{14}$

(3) $2\sqrt{14}$

(4) $2\sqrt{7}$

**Ans. (3)**

**Sol.** $\vec{b} = \vec{a} \times (\hat{i} - 3\hat{k})$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 0 & -2 \end{vmatrix} = 2\hat{i} + 8\hat{j} + \hat{k}$

$\vec{c} = \vec{b} \times \hat{k} = 8\hat{i} - 2\hat{j}$

$\vec{c} - 2\hat{j} = 8\hat{i} - 4\hat{j}$

Projection of $(\hat{i} - 2\hat{j})$ on $\vec{a}$

$(\vec{c} - 2\hat{j}).\hat{a} = \frac{\langle 8, -4, 0 \rangle . \langle 3, -1, 2 \rangle}{\sqrt{14}}$

$= \frac{28}{\sqrt{14}} = 2\sqrt{14}$

**15.** For some a, b, let

$f(x) = \begin{vmatrix} a + \frac{\sin x}{x} & 1 & b \\ a & 1 + \frac{\sin x}{x} & b \\ a & 1 & b + \frac{\sin x}{x} \end{vmatrix}, \quad x \neq 0,$

$\lim_{x \to 0} f(x) = \lambda + \mu a + \nu b$. Then $(\lambda + \mu + \nu)^2$ is equal to:

(1) 25 (2) 9

(3) 36 (4) 16

**Ans. (4)**

**Sol.** $\lim_{x \to 0} f(x) = \begin{vmatrix} a+1 & 1 & b \\ a & 1+1 & b \\ a & 1 & b+1 \end{vmatrix}$

$= (a+1)(2(b+1) - b) + 1(ab - a(b+1)) + ba$

$= (a+1)(b+2) - a + ab$

$= b + a + 2 = \lambda + \mu a + \nu b$

$\lambda = 2, \mu = 1, \nu = 1 \Rightarrow (\lambda + \mu + \nu)^2 = 16$

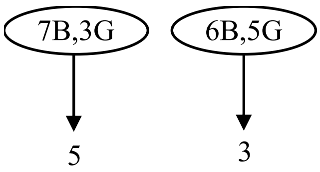
**16.** Group A consists of 7 boys and 3 girls, while group B consists of 6 boys and 5 girls. The number of ways, 4 boys and 4 girls can be invited for a picnic if 5 of them must be from group A and the remaining 3 from group B, is equal to:

(1) 8575 (2) 9100

(3) 8925 (4) 8750

**Ans. (3)**

**Sol.**




C-I (3G & 2B) & (1G & 2B)

C-II (2G & 3B) & (2G & 1B)

C-III (1G & 4B) & (3G & 0B)

Total = C-I + C-II + C-III

$= {}^7C_2 . {}^3C_3 . {}^6C_2 . {}^5C_1 + {}^7C_3 . {}^3C_2 . {}^6C_1 {}^5C_2 + {}^7C_4 . {}^3C_1 . {}^6C_0 . {}^5C_3$

= 8925

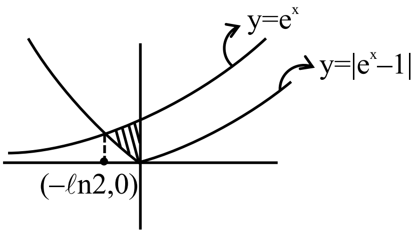
**17.** The area of the region enclosed by the curves $y = e^x$, $y = |e^x - 1|$ and y-axis is:

(1) $1 + \log_e 2$ (2) $\log_e 2$

(3) $2 \log_e 2 - 1$ (4) $1 - \log_e 2$

**Ans. (4)**

**Sol.**




For Area $\int_{-\ell n2}^{0} \left[e^x - \left(1 - e^x\right)\right] dx$

$\int_{-\ell n2}^{0} \left(2e^x - 1\right) dx = \left[2e^x - x\right]_{-\ell n2}^{0}$

$= (2 - (1 + \ell n2))$

$= 1 - \ell n2$

**18.** The number of real solution(s) of the equation $x^2 + 3x + 2 = \min\{|x - 3|, |x + 2|\}$ is :

(1) 2

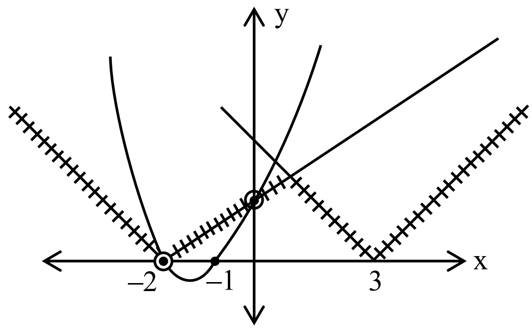
(2) 0

(3) 3

(4) 1

**Ans. (1)**

**Sol.**




Only 2 solutions.

**19.** Let $A = [a_{ij}]$ be a square matrix of order 2 with entries either 0 or 1. Let E be the event that A is an invertible matrix. Then the probability P(E) is :

(1) $\frac{5}{8}$ (2) $\frac{3}{16}$

(3) $\frac{1}{8}$ (4) $\frac{3}{8}$

**Ans. (4)**

**Sol.** C-I $\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \rightarrow 4$ ways

C-II $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ & $\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \rightarrow 2$ ways

$P = \frac{\text{favourable}}{\text{total}} = \frac{6}{16} = \frac{3}{8}$

**20.** If the equation of the parabola with vertex $V\left(\frac{3}{2}, 3\right)$ and the directrix $x + 2y = 0$ is $\alpha x^2 + \beta y^2 - \gamma xy - 30x - 60y + 225 = 0$, then $\alpha + \beta + \gamma$ is equal to:

(1) 6

(2) 8

(3) 7

(4) 9

**Ans. (4)**

**Sol.** Equation of axis $y - 3 = 2\left(x - \frac{3}{2}\right)$

$y - 2x = 0$

foot of directrix

$y - 2x = 0$

& $\Rightarrow (0, 0)$

$2y + x = 0$

Focus = (3, 6)

$PS^2 = PM^2$

$(x-3)^2 + (y-6)^2 = \left(\frac{x+2y}{\sqrt{5}}\right)^2$

$4x^2 + y^2 - 4xy - 30x - 60y + 225 = 0$

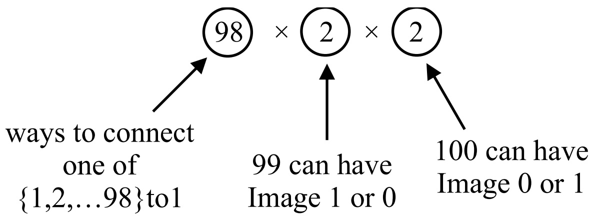
$\Rightarrow \alpha = 4, \beta = 1, \gamma = 4 \Rightarrow \alpha + \beta + \gamma = 9$

**SECTION-B**

**21.** Number of functions $f : \{1, 2, \ldots, 100\} \to \{0, 1\}$, that assign 1 to exactly one of the positive integers less than or equal to 98, is equal to ______.

**Ans. (392)**

**Sol.**

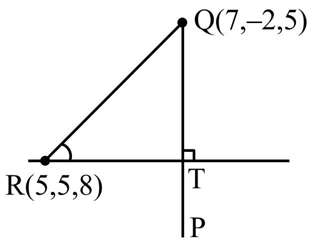



392 Ans.

**22.** Let P be the image of the point Q(7, –2, 5) in the line $L : \frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$ and R(5, p, q) be a point on L. Then the square of the area of ΔPQR is ____

**Ans. (957)**

**Sol.**




Let R(2λ + 1, 3λ – 1, 4λ)

2λ + 1 = 5

λ = 2

R(5, 5, 8)

let T(2λ + 1, 3λ – 1, 4λ)

$\overrightarrow{QT} = (2\lambda - 6)\hat{i} + (3\lambda + 1)\hat{j} + (4\lambda - 5)\hat{k}$

$\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\overrightarrow{QT} \cdot \vec{b} = 0$

4λ – 12 + 9λ + 3 + 16λ – 20 = 0

λ = 1

T(3, 2, 4)

$QT = \sqrt{33}$ $RT = \sqrt{29}$

$(\text{area of } \Delta PQR)^2 = \left(\frac{1}{2}\sqrt{29}.2\sqrt{33}\right)^2$

= 957

**23.** Let y = y(x) be the solution of the differential equation $2\cos x \frac{dy}{dx} = \sin 2x - 4y \sin x$, $x \in \left(0, \frac{\pi}{2}\right)$. If $y\left(\frac{\pi}{3}\right) = 0$, then $y'\left(\frac{\pi}{4}\right) + y\left(\frac{\pi}{4}\right)$ is equal to ____.

**Ans. (1)**

**Sol.** $\frac{dy}{dx} + 2y\tan x = \sin x$

$\text{I.F.} = e^{2\int \tan x\, dx} = \sec^2 x$

$y\sec^2 x = \int \frac{\sin x}{\cos^2 x} dx$

$= \int \tan x \sec x\, dx$

= secx + C

C = –2

$y = \cos x - 2\cos^2 x$

$y\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} - 1$

y′ = –sinx + 4cosx sinx

$y'\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}} + 2$

$y'\left(\frac{\pi}{4}\right) + y\left(\frac{\pi}{4}\right) = 1$

**24.** Let $H_1 : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $H_2 : -\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$ be two hyperbolas having length of latus rectums $15\sqrt{2}$ and $12\sqrt{5}$ respectively. Let their eccentricities be $e_1 = \sqrt{\frac{5}{2}}$ and $e_2$ respectively. If the product of the lengths of their transverse axes is $100\sqrt{10}$, then $25e_2^2$ is equal to ______

**Ans. (55)**

**Sol.** $\frac{2b^2}{a}=15\sqrt{2}$

$1+\frac{b^2}{a^2}=\frac{5}{2}$

$a=5\sqrt{2}$

$b=5\sqrt{3}$

$\frac{2A^2}{B}=12\sqrt{5}$

$2a.2B = 100\sqrt{10}$

$2.5\sqrt{2}.2B=100\sqrt{10}$

$B = 5\sqrt{5}$

$A = 5\sqrt{6}$

$e_2^2=1+\frac{A^2}{B^2}$

$=1+\frac{150}{125}$

$e_2^2=1+\frac{30}{25}$

$25e_2^2=55$

**25.** If $\int\frac{2x^2+5x+9}{\sqrt{x^2+x+1}}dx=x\sqrt{x^2+x+1}+\alpha\sqrt{x^2+x+1}+\beta\log_e\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|+C$, where C is the constant of integration, then $\alpha + 2\beta$ is equal to ___

**Ans.** **(16)**

**Sol.** $2x^2 + 5x + 9 = A(x^2 + x + 1) + B(2x + 1) + C$

$A = 2 \qquad B = \frac{3}{2} \qquad C = \frac{11}{2}$

$2\int\sqrt{x^2+x+1}\,dx+\frac{3}{2}\int\frac{2x+1}{\sqrt{x^2+x+1}}dx+\frac{11}{2}\int\frac{dx}{\sqrt{x^2+x+1}}$

$2\int\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dx+3\sqrt{x^2+x+1}+\frac{11}{2}\int\frac{dx}{\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}}$

$2\left(\frac{x+\frac{1}{2}}{2}\sqrt{x^2+x+1}+\frac{3}{8}\ell n\left(x+\frac{1}{2}+\sqrt{x^2+x+1}\right)\right)+3\sqrt{x^2+x+1}$

$+\frac{11}{2}\ell n\left(x+\frac{1}{2}+\sqrt{x^2+x+1}\right)+C$

$\alpha=\frac{7}{2} \qquad \beta=\frac{25}{4}$

$\alpha + 2\beta = 16$